CALCULATION OF THE FLOW IN THE MAIN SECTION OF A SUPERSONIC JET WITH THE NOZZLE END TAKEN INTO ACCOUNT

P. A. Neshcheret, E. A. Kapustin, and O. É. Shlik UDC 532.525.2:534.83

It is well known that the acoustic environment in the surrounding space, including the location and sizes of acoustic shields reflecting the intrinsic radiation of the jet, has no significant effect on the propagation of a submerged supersonic jet, which leads to the onset under certain conditions of a resonance effect, the so-called acoustic feedback. The conditions for the onset of acoustic feedback are the presence in the noise spectrum of a discrete tone isolated in amplitude [1-3], which is related to the regime of jet outflow from the nozzle (the Mach number of the nozzle M_a and the noncalculability of the outflow $n = p_a/p_{in}$) and also the presence of acoustic shields which reflect the radiation back into the jet [4]. The nozzle end is the reflecting shield encountered most often and located most closely to the sensitive zone of the jet.

The effect of the diameter of the nozzle end on the dynamic characteristics of a jet has been investigated previously [5, 6], and formulas have been obtained for determination of the excess drag pressure on the jet axis in the main turbulent section $\bar{p} = p_{0d}/p_{0\infty}$ (p_{0d} is the pressure corresponding to a specific end shield with diameter d_e, and $p_{0\infty}$ is the pressure corresponding to an infinite end shield (flat wall)). The scheme for construction of the curve of variation of p_0 as a function of $\bar{d} = d_e/d_a$ which has been adopted in [5] is given in Fig. 1 ($M_a = 1.9$, $n = 0.8$, and the aperture angle of the nozzle $\gamma = 40°$).

The formulas are obtained in the form

$$\bar{p}_0 = 1 + B,$$

where

$$B = 0.5\left[A_u e^{-K_u x}\left(1+\cos\frac{2\pi x^2}{\lambda_i}\right) - A_l e^{-K_l x}\left(1-\cos\frac{2\pi x^2}{\lambda_i}\right)\right]; \quad (1)$$

$$A_u = A_\Sigma\left[1 - 0.7\left(\frac{n-n_a}{\Delta n}\right)^2\right]; \quad A_\Sigma = 1.6\,(M_a-1)^{0.5}\Delta n\cos^2\gamma\cdot\sin\frac{n-n_a}{\Delta n}\pi;$$

γ is the aperture angle of the supersonic part of the nozzle at the cutoff, n_a is the smallest value of the noncalculability of onset of acoustic resonance, $x = \sqrt{\bar{d}-1}$, and λ_i is the length of the i-th wave of pressure variation associated with the wavelength of the discrete radiation. The values of λ_i and the remaining quantities, whose meaning is clear from Fig. 1, are determined using the formulas of [6]. A refined value of n_a and the range of noncalculabilities of jet outflow Δn in which the effect of acoustic feedback is manifested can be taken from [7]; the formulas are suitable for the entire range of existence of acoustic feedback (both overexpanded and underexpanded outflow).

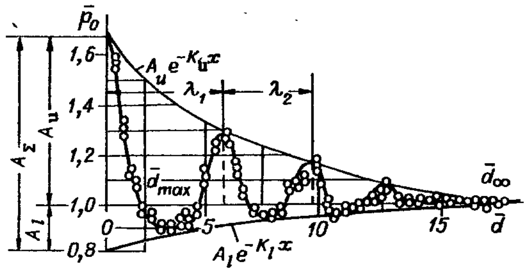

Fig. 1

Dnepropetrovsk. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 81-85, September-October, 1984. Original article submitted July 20, 1983.

 0021-8944/84/2505-0728$08.50

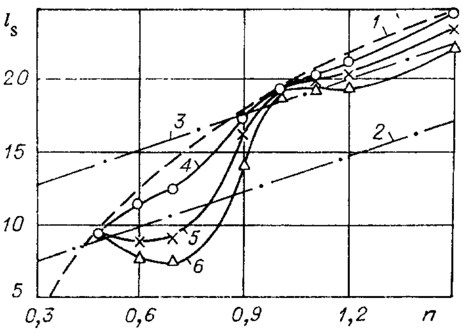

Fig. 2

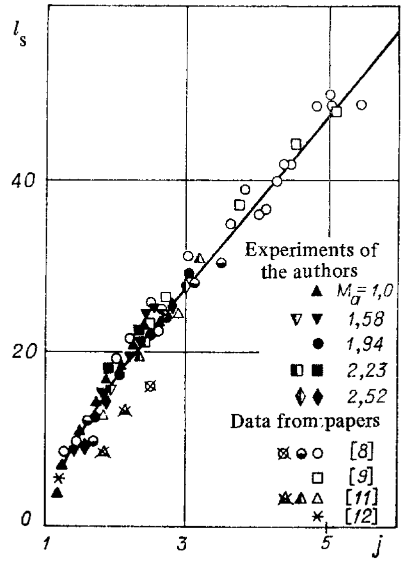

Fig. 3

The formulas obtained permit determining the effect of different parameters on the pressure in the subsonic part of the jet, but they do not permit calculating the flow field. Knowledge of the position of the sonic cross section of the jet l_s and the turbulence coefficient of the flow $c = dr/dl$, where r is the jet radius, is desirable for the calculation.

In this connection investigations to find the dependence of the coordinate of the sonic cross section $\bar{l}_s = l_s/d_a$ on the diameter of the nozzle end were performed similarly to the determination of $\bar{p}_0$.

The position of the sonic cross section with a disk of constant diameter mounted flush with the nozzle cutoff was determined as a function of the noncalculability of the outflow by moving a total-head tube along the jet axis. The procedure for performing the experiments and the region of variation of the parameters of the investigation correspond to those given in [5, 6].

An example of the experimental results is given in Fig. 2, where $M_a = 1.9$, curve 1 — according to [9] and formula (2), curve 2 — according to [10] with $\beta = 0$, curve 3 — according to [10] with $\beta = 3$, curve 4 — experiment with $\bar{d} = 1.2$, curve 5 — $\bar{d} = \bar{d}_{max} = 3.6$, and curve 6 — $\bar{d}_\infty = 18$. It is evident that the variation of the position of the sonic cross section $\bar{l}_s$ as a function of n and $\bar{d}$ is similar to the pressure variation described in [3, 5, 6]. The smallest length of the supersonic flow section is observed at an end diameter equal to the diameter of maximum acoustic action d_{max}. The largest length is achieved for a nozzle edge of zero thickness $\bar{d} \to d_i = 1$, i.e., for an ideal acoustically isolated nozzle.

The results obtained were compared with the data of [8], [9], and others. The authors' own experiments and the results of other papers generalized in terms of the parameter kM_a, where k is the adiabatic index, are given in [8]; then the product of $\bar{l}_s$ by a function of this parameter depends only on the noncalculability n. However, as has been shown in [8], with this kind of analysis points with n = 1 comprise an exception — the length of the supersonic section is higher in these cases by 40-60% than according to the equation presented in [10] on the basis of the results of [8]. It has been confirmed in [9] on the basis of their own results and the data of [8] that the coordinate of the sonic cross section is generalized if one expresses $\bar{l}_s$ with flow noncalculabilities n > 2 as a function of the parameter $j = M_a\sqrt{nk}$.

Experiments on the effect on l_s of the nozzle end have shown that the generalization of [9] can be extended also to the region n < 2 (curve 1, Fig. 2) but only for a nozzle with a sharp edge ($l_s = l_i$). The points for n = 1 obtained both by us and in [8, 11] are also subject to generalization (Fig. 3, the half-darkened symbols). Only the results of [8] for $M_a = 1.5$ and n = 2 and of [11] for $M_a = 2.0$ and n = 0.6 and 0.8 stand apart from the dependence (the crossed-out symbols). The regimes indicated above correspond, according to [7], to the greatest influence of acoustic feedback. This offers a basis for assuming either that the values given are obtained with nozzles having a wide edge or that a reflector of acoustic radiation from the jet was located near the nozzle cutoff. In the case n = 1 the results of these same authors show no deviations from the generalizing dependence, since the effect of acoustic feedback at $n \approx 1$ decreases significantly (see Fig. 2) due to a weakening of the shock waves in the jet in the calculated outflow regime.

The generalizing curve can be described by the equation

$$\bar{l}_i = 9.5(j - 1.05/j^{2.7}). \tag{2}$$

For $n > 3$-3.5 Eq. (2) degenerates into a straight line similar to that indicated in [9] and confirming the self-similarity of the relative longitudinal dimensions of the jets. The determination of the length of the supersonic section $\bar{l}_s$ using the formula written in [10] gives results which differ from the experiments (see Fig. 2), evidently due to a nonexpansion of the effect of acoustic self-excitation of the jet, which is, however, specified there by the introduction of a coefficient β which varies from 0 to 3 in jets with regular reflection of condensation jumps away from the axis (the dependence itself is not given).

The effect of d_e on l_s turned out, as analysis of the experiments has shown, to be similar to the effect on p_0, which permitted representing the dependence $l_{sd}/l_{s\infty}$ as a function of B, where l_{sd} is the coordinate of the sonic cross section for a nozzle with an end diameter d_e and $l_{s\infty}$ is the coordinate for $d_e = \infty$ in a form similar to formula (1):

$$\bar{l}_s = 1 + k_s B. \tag{3}$$

The value of the coefficient k_s for the region of developed acoustic feedback, for which formulas (1) are obtained, can be assumed to be constant and equal to 0.8.

Then

$$\bar{l}_s = \bar{l}_i \frac{1 + 0.8B}{1 + 0.8A_u}, \tag{4}$$

where A_u and B are determined from the corresponding dependences and $\bar{l}_i$ is determined from the generalized plot (see Fig. 3) or from Eq. (2).

In order to determine the turbulence coefficient c in the very same range of variation of the parameters of the investigation, experiments were performed which are similar to the determination of $\bar{l}_s$. The procedure and examples of the experimental determination of c are expounded in [5]. The interpretation and analysis of the experimental data have permitted writing, similarly to (4), with satisfactory accuracy for a flat end shield

$$c = c_0 \sqrt{(1 + B)/(1 + A_u)}, \tag{5}$$

where c_0 is the characteristic of the turbulence at the nozzle cutoff (usually taken to be 0.06-0.07 for free jets).

The results obtained permit calculating the main section of the flow with account taken of the effect of the acoustic radiation of the jet, which interacts with the nozzle end. To this end one can make use of the procedure of [13] of replacing the main section of the actual uncalculated supersonic jet with the main section of a fictitious jet for which the outflow from a fictitious nozzle with d_{af} is calculated and sonic ($\lambda_{m0} = V_0/a = 1$) and also the diameter of the transition section d_{tf} is equal to the diameter of the sonic cross section of the actual jet d_s. The flow in the core of a fictitious jet with length x_{tf} is a potential flow. The replacement is accomplished on the assumption of self-similarity of the velocity profile in the sonic cross section of the uncalculated jet under discussion and an isobaric nature of the jet in this cross section. Then the dimensionless polar distance of the point under discussion located a distance l from the nozzle cutoff should be determined as

$$\frac{x_f}{d_{af}} = \frac{x_{tf}}{d_{af}} + (\bar{l} - \bar{l}_s)\frac{d_{tf}}{d_{af}}\frac{1}{\bar{d}_s}. \tag{6}$$

With the help of (6) one can obtain the distribution of the dimensionless velocity along the axis:

$$\lambda_m = \left(1 + 5.67\, c \frac{\bar{l} - \bar{l}_s}{\bar{d}_s}\right)^{-1},$$

where $\bar{l}_s$ is determined from (2), c from (5), and $\bar{d}_s$ using the formula from [10] obtained on the basis of conservation of momentum along the jet axis:

$$\bar{d}_s = 2.72\sqrt{nM_a^2 + \frac{n-1}{k}},$$

which is in good agreement with our experimental data.

Thus it is possible to take account of the effect of the edge thickness of an actual nozzle in terms of the acoustic feedback on the dynamic characteristics of a supersonic jet.

LITERATURE CITED

1. V. M. Mamin, "The problem of the mechanism of radiation of a discrete tone by supersonic jets," in: Investigations in Vibrational Heating and Related Problems [in Russian], Kaz. Univ., Kazan' (1974).
2. A. V. Antsupov and V. G. Pimshtein, "The radiation of a discrete tone by a supersonic jet flowing out of a conical nozzle," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 1 (1975).
3. A. N. Antonov, S. P. Shalaev, and M. Ya. Yudelovich, "The effect of the discrete component of acoustic oscillations on the flow in an uncalculated supersonic jet," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 3 (1973).
4. A. N. Antonov, S. P. Shalaev, and M. Ya. Yudelovich, "The effect of acoustical reflectors on the discrete component in the noise spectrum of a supersonic uncalculated jet," Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza, No. 6 (1977).
5. I. V. Belov, E. A. Kapustin, P. A. Neshcheret, and O. É. Shlik, "The effect of the end of a supersonic nozzle on the flow in the subsonic part of an overexpanded jet," in: Turbulent Jet Flows [in Russian], In-t Termofiziki i Elektrofiziki AN Est. SSR, Tallin (1979).
6. E. A. Kapustin, P. A. Neshcheret, and O. É. Shlik, "Characteristics of the outflow of an overexpanded jet from a nozzle with an edge of finite thickness," in: The Aerodynamics of Engineering Processes [in Russian], Nauka, Moscow (1981).
7. E. A. Kapustin, I. A. Lentsov, P. A. Neshcheret, and O. É. Shlik, "Acoustic feedback of supersonic jets for different flow regimes," In: Hydroaeromechanics and Elasticity Theory [in Russian], No. 28, Dnepropetrovsk (1981).
8. D. Shiri and D. Sebold, "The length of the supersonic core of high-velocity jets," Raketn. Tekh. Kosmonavtika, 5, No. 11 (1967).
9. G. F. Glotov and M. I. Feiman, "An investigation of the parameters of axisymmetric underexpanded gas jets flowing out into a submerged space," Uchen. Zap. TsAGI, 2, No. 4 (1971).
10. A. V. Antsupov, "An investigation of the parameters of an uncalculated supersonic gas jet," Zh. Tekh. Fiz., 44, No. 2 (1974).
11. A. V. Antsupov and V. I. Blagosklonov, "The structure of a supersonic jet flowing out into a submerged space," Tr. TsAGI, No. 1781 (1976).
12. V. N. Glaznev and Sh. Suleimanov, Gas Dynamic Parameters of Weakly Underexpanded Jets [in Russian], Nauka, Novosibirsk (1980).
13. V. E. Davidson, "The penetration depth of a gas jet into a liquid upon purging from above with account taken of wave and viscous losses," Izv. Vyssh. Uchebn. Zaved., Chernaya Metallurgiya, No. 1 (1977).